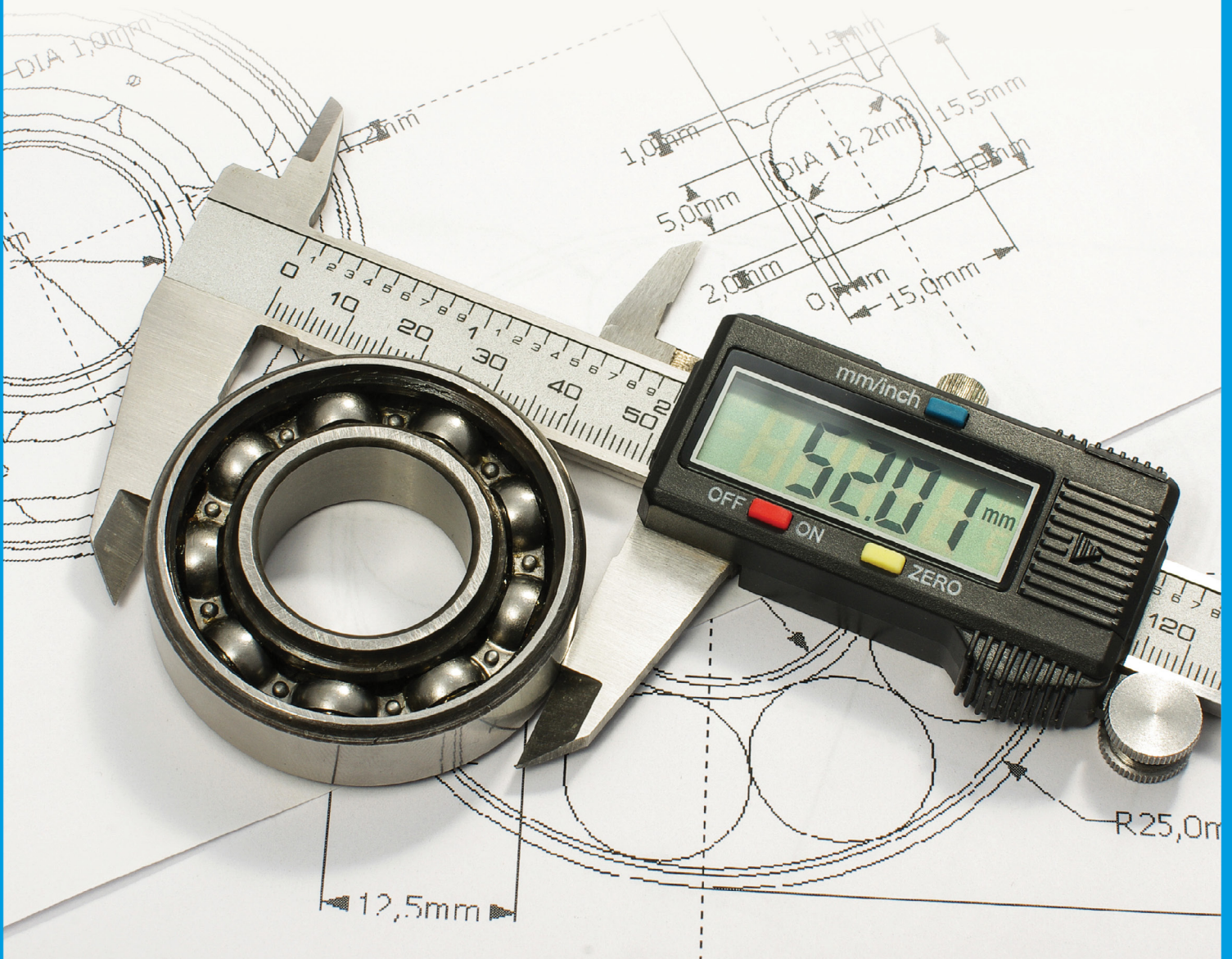


8TH EDITION

MATHEMATICS

for MACHINE TECHNOLOGY

John C. Peterson
Robert D. Smith



MATHEMATICS *for* MACHINE TECHNOLOGY

EIGHTH EDITION

John C. Peterson
Robert D. Smith



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Preface

“I can’t think of any job in my sheet metal fabrication company where math is not important. If you work in a manufacturing facility, you use math every day; you need to compute angles and understand what happens to a piece of metal when it’s bent to a certain angle.”¹—TRACI TAPANI, WYOMING MACHINE

Mathematics for Machine Technology is written to overcome the often mechanical “plug in” approach found in many trade-related mathematics textbooks. An understanding of mathematical concepts is stressed in all topics ranging from general arithmetic processes to oblique trigonometry, compound angles, and numerical control.

Both content and method are those that have been used by the authors in teaching applied machine technology mathematics classes for apprentices in machine, tool-and-die, and tool design occupations. Each unit is developed as a learning experience based on preceding units—making prerequisites unnecessary.

Presentation of basic concepts is accompanied by realistic industry-related examples and actual industrial applications. The applications progress from the simple to those with solutions that are relatively complex. Many problems require the student to work with illustrations such as are found in machine technology handbooks and engineering drawings.

Great care has been taken in presenting explanations clearly and in providing easy-to-follow procedural steps in solving exercise and problem examples. The book contains a sufficient number of exercises and problems to permit the instructor to selectively plan assignments.

An analytical approach to problem solving is emphasized in the geometry, trigonometry, compound angle, and numerical control sections. This approach is necessary in actual practice in translating engineering drawing dimensions to machine working dimensions. Integration of algebraic and geometric principles with trigonometry by careful sequence and treatment of material also helps the student in solving industrial applications. The Instructor’s Guide provides answers and solutions for all problems.

A majority of instructors state that their students are required to perform basic arithmetic operations on fractions and decimals prior to calculator usage. Thereafter, the students use the calculator almost exclusively in problem-solving computations. Calculator instructions and examples have been updated and expanded in this edition. The scientific calculator and the MACHINIST CALC PRO 2 are introduced in the Preface. Extensive calculator instruction and examples are given directly following the units on fractions and mixed numbers and the units on decimals. Further calculator instruction and examples are given throughout the text wherever calculator applications are appropriate to the material presented.

¹ Source: Thomas Friedman, “If You’ve Got the Skills, She’s Got the Job,” *New York Times*, November 17, 2012, accessed November 18, 2012, <http://www.nytimes.com/2012/11/18/opinion/sunday/Friedman-You-Got-the-Skills.html>

Many students and workers use tablet computers. One of the advantages of a tablet is that apps for scientific calculators, the MACHINIST CALC PRO 2, and spreadsheets can be installed on a tablet. This text includes both spreadsheet and calculator instructions.

As with the calculator, extensive spreadsheet instruction and examples are given directly following the unit on using calculators with fractions and mixed numbers and the unit on using calculators with decimals. Further spreadsheet instruction and examples are given throughout the text wherever spreadsheet applications are appropriate.

Changes from the previous edition have been made to improve the presentation of topics and to update material.

A survey of instructors using the seventh edition was conducted. Based on their comments and suggestions, changes were made. The result is an updated and improved eighth edition that includes the following revisions:

Four major changes were made in this edition.

- Spreadsheet instruction was added for tablets like the iPad, Microsoft Surface, and Samsung Galaxy. Many workers are using laptops on the job because of their convenience and portability. While tablets are bulkier than calculators, they have the advantage of having both spreadsheet and calculator apps, so a user needs only one machine.
- A unit on introductory G- and M-codes for CNC programming was added. This unit is only designed to introduce students to the G-codes that prepare a machine to engage in a particular mode for machining and the M-codes that are used to turn on and off miscellaneous functions.
- Selected calculator instruction was included for the MACHINIST CALC™ PRO 2, Model 4088. This particular calculator can be used with material and tool settings combined with DOC (depth of cut) and WOC (width of cut) to solve speed and feed calculations for face, end, or slot milling plus turning, drilling and boring. It also gives step-saving drill and thread chart lookups, right triangle solutions, bolt-circle patterns, etc. It does have some limitations, particularly when working with fractions, and these limitations are pointed out in the text.
- Many existing calculator instructions were deleted. Students are familiar with calculators and have probably been using them since elementary school and should not need basic instruction in calculator usage. Since most new scientific calculators use a “natural display,” much of the duplicate instructions for calculator keystrokes has also been deleted.
- A supplement was prepared that includes MACHINIST CALC™ PRO 2, Model 4088 instructions for all places in the text where calculators are used. The supplement also has instructions that show how the calculator can be used for some topics that are not part of the text.

ABOUT THE AUTHORS

John C. Peterson is a retired professor of mathematics at Chattanooga State Community College, Chattanooga, Tennessee. Before he began teaching, he worked in the manufacturing industry. He has taught at the middle school, high school, two-year college, and university levels. Dr. Peterson is the author or coauthor of four other Cengage Learning books: *Introductory Technical Mathematics* (with Robert D. Smith), *Technical Mathematics*, *Technical Mathematics with Calculus*, and *Math for the Automotive Trade* (with William J. deKryger). He was a member of the four-person team that revised and interpreted the

two-year college portion of the 2015 College Board of Mathematical Sciences cross-sectional survey of undergraduate mathematical science programs. In addition, he has had over 90 papers published in various journals, has given over 200 presentations, and has served as a vice president of the American Mathematical Association of Two-Year Colleges.

Robert D. Smith was Associate Professor Emeritus of Industrial Technology at Central Connecticut State University, New Britain, Connecticut. Mr. Smith had experience in the manufacturing industry as tool designer, quality control engineer, and chief manufacturing engineer. He also taught applied mathematics, physics, and industrial materials and processes on the secondary technical school level and machine technology applied mathematics for apprentices in machine, tool-and-die, and tool design occupations. He was the author of *Technical Mathematics 4e*, also published by Cengage.

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In addition, the publisher and author acknowledge Linda Willey for her tireless commitment to the technical review of the text, examples, applications, answers, and solutions.

INTRODUCTION TO THE SCIENTIFIC CALCULATOR



A scientific calculator is to be used in conjunction with the material presented in this textbook. Complex mathematical calculations can be made quickly, accurately, and easily with a scientific calculator.

Although most functions are performed in the same way, there are some variations among different makes and models of scientific calculators. In this book, generally, where there are two basic ways of performing a function, or sequencing, both ways are shown. However, not all of the differences among the various makes and models of calculators can be shown. It is very important that you become familiar with the operation of your scientific calculator. An owner's manual or user's guide is included with the purchase of a scientific calculator; it explains the essential features and keys of the specific calculator, as well as providing information on the proper use. *It is important that the owner's manual or user's guide be studied and referred to whenever there is a question regarding calculator usage.* Also, information can be obtained from the manufacturer's Internet website, which is often listed in the user's guide.

Most scientific calculator keys can perform more than one function. Depending on the calculator, generally the **2nd** key or **SHIFT** key enable you to use alternate functions. The alternate functions are marked above the key. Alternate functions are shown and explained in the book where their applications are appropriate to specific content.

INTRODUCTION TO THE MACHINIST CALC PRO 2

The MACHINIST CALC PRO 2 is more than just a calculator. It is a dedicated mathematics and reference tool that provides a complete assortment of machine shop solutions. It has more complete thread data than the *Machinery's Handbook*, including step-saving thread and drill size chart lookups, speeds and feeds, right triangle solutions, and bolt patterns. It also provides hundreds of fast, precise machining-specific solutions for turning, drilling, boring, and face, end, and slot milling. Built-in tables for 20 materials, six processes and three tools means that you spend much less time looking up your most needed calculations on charts, in books, or on the Internet and more time machining.

Although most functions are performed in the same way on the MACHINIST CALC PRO 2 as on other calculators, there are some important differences. A *User's Guide* is included with the purchase of the MACHINIST CALC PRO 2 and explains the essential features and keys of the calculator, as well as providing detailed information on proper use. *It is essential that the User's Guide be studied and referred to whenever there is a question regarding calculator use.* The Calculated Industries Website² has some video tutorials that show how to use the calculator for some specific needs. In addition, apps for the iPhone, iPad, and Android mobile devices can be downloaded from the webpage.

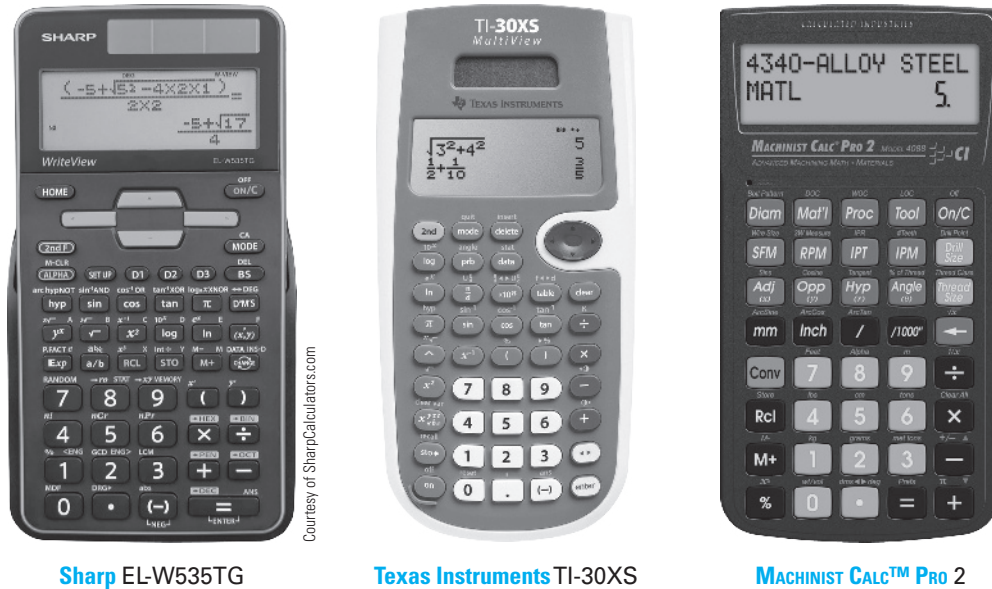
Because the MACHINIST CALC PRO 2 is designed for machinists, it cannot perform all the mathematics functions needed for this text. However, in many cases, examples show how to use the MACHINIST CALC PRO 2 in these situations.

DECISIONS REGARDING CALCULATOR USE

The exercises and problems presented throughout the text are well suited for solutions using a calculator. However, it is felt that decisions regarding calculator usage should be left to the discretion of the course classroom or shop instructor. The instructor best knows the unique learning environment and objectives to be achieved by the students in a course. Judgments should be made by the instructor as to the degree of emphasis to be placed on calculator applications, when and where a calculator is to be used, and the selection of specific problems for solution by calculator. Therefore, exercises and problems in this text are *not* specifically identified as calculator applications.

Calculator instruction and examples of the basic operations of addition, subtraction, multiplication, and division of fractions are presented in Unit 7. They are presented for decimals in Unit 17. Further calculator instruction and examples of mathematics operations and functions are given throughout the text wherever calculator applications are appropriate to the material presented.

² <http://www.calculated.com/46/prd377/Machinist-Calc-Pro-2.html#VG-e8r6Ex2k>



Sharp EL-W535TG

Texas Instruments TI-30XS

MACHINIST CALC™ Pro 2

INTRODUCTION TO SPREADSHEETS

A spreadsheet is a computer application that simulates a paper accounting worksheet. It displays multiple cells in a grid formed by rows and columns. Each cell in the grid contains text, a numeric value, or a formula. A formula defines how the content of that cell is to be calculated from the contents of any other cell (or combination of cells) each time any cell is updated.

The most popular spreadsheet is Microsoft's Excel, which works on the Macintosh iPad, Windows-based tablets like the Surface and Asus Transformer Book Flip, and Android-based machines like the Google Pixel, Samsung Galaxy Tab, and Sony Xperia. Some other programs similar to Excel are:

- **Numbers**, Apple Inc.'s spreadsheet software, is part of iWork.
- **Google Sheets** is free for all Google account owners. It is a cloud-based program that can be used on a phone, tablet, or computer.
- **Apache OpenOffice Calc** is a freely available, open-source program.
- **LibreOffice**, like Apache Open Office, was derived from the original Openoffice.org program.
- **Bime Analytics** is a new way of expressing and analyzing data in a more presentable display.
- **ThinkFree** is a free Excel alternative that is quite similar to Google Drive Apps and lets you create several different types of online documents.
- **Gnumeric** is a simple open-source spreadsheet application that has all the basic Excel features. Like many other free Excel alternatives, it's a fine tool for less complicated data analysis.
- **BIRT Spreadsheet**, formerly named Actuate e.Spreadsheet, is a downloadable application when you need to automate and centralize spreadsheet production, maintenance, archiving, and security.
- **Zoho Sheet** is another cloud-based free Excel alternative that empowers your productivity and enables working in an Excel-like environment, creating new spreadsheets and editing documents written in other spreadsheet applications.

The illustrations in this text were all made using Excel on an iPad.

Common Fractions and Decimal Fractions



UNIT 1

Introduction to Common Fractions and Mixed Numbers

OBJECTIVES After studying this unit you should be able to

- Express fractions in lowest terms.
- Express fractions as equivalent fractions.
- Express mixed numbers as improper fractions.
- Express improper fractions as mixed numbers.

Most measurements and calculations made by a machinist are not limited to whole numbers. Dimensions are sometimes given as fractions and certain measuring tools are graduated in fractional units. The machinist must be able to make calculations using fractions and to measure fractional values.

FRACTIONAL PARTS

A *fraction* is a value that shows the number of equal parts taken of a whole quantity or unit. The symbols used to indicate a fraction are the bar (—) and the slash (/).

Line segment AB as shown in Figure 1-1 is divided into 4 equal parts.

$$1 \text{ part} = \frac{1 \text{ part}}{\text{total parts}} = \frac{1 \text{ part}}{4 \text{ parts}} = \frac{1}{4} \text{ of the length of the line segment.}$$

$$2 \text{ parts} = \frac{2 \text{ parts}}{\text{total parts}} = \frac{2 \text{ parts}}{4 \text{ parts}} = \frac{2}{4} \text{ of the length of the line segment.}$$

$$3 \text{ parts} = \frac{3 \text{ parts}}{\text{total parts}} = \frac{3 \text{ parts}}{4 \text{ parts}} = \frac{3}{4} \text{ of the length of the line segment.}$$

$$4 \text{ parts} = \frac{4 \text{ parts}}{\text{total parts}} = \frac{4 \text{ parts}}{4 \text{ parts}} = \frac{4}{4} = 1, \text{ or unity (four parts make up the whole).}$$

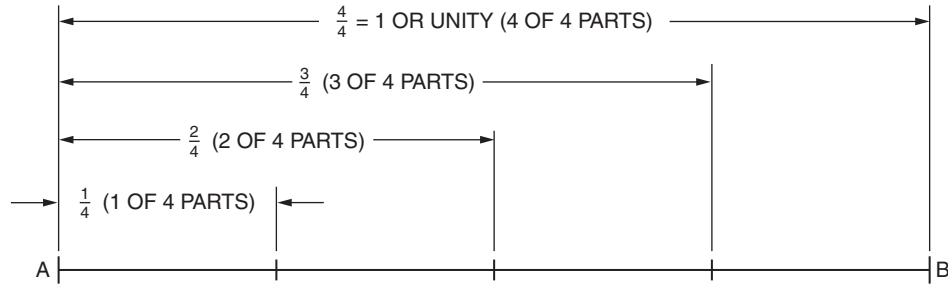


FIGURE 1-1

Each of the 4 equal parts of the line segment AB in Figure 1-2 is divided into eight equal parts. There is a total of 4×8 , or 32, parts.

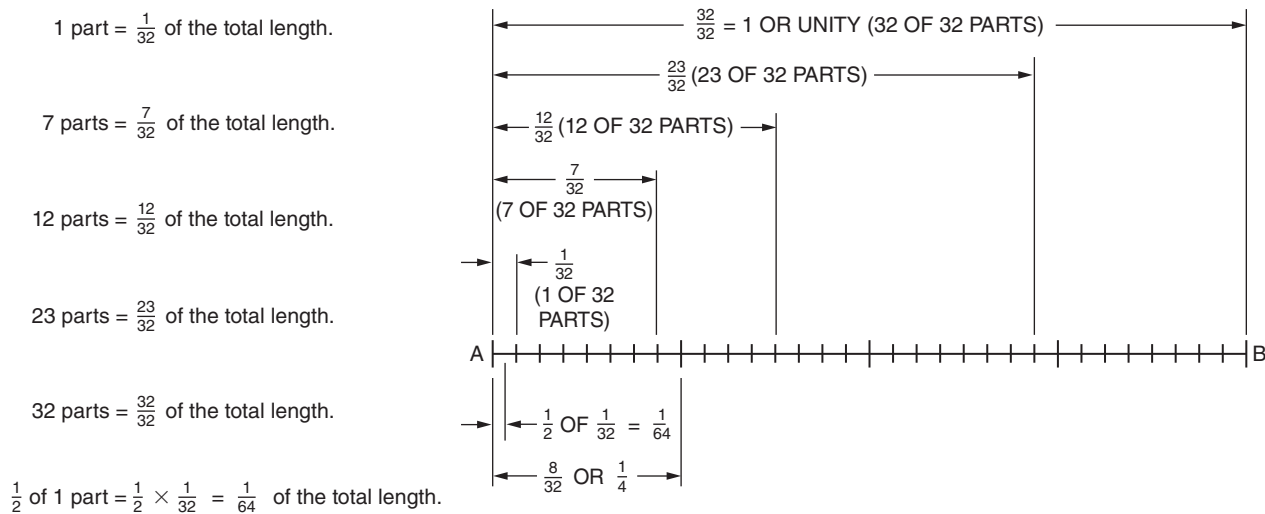


FIGURE 1-2

Note: 8 parts = $\frac{8}{32}$ of the total length and also $\frac{1}{4}$ of the total length. Therefore, $\frac{8}{32} = \frac{1}{4}$.

DEFINITIONS OF FRACTIONS

A *fraction* is a value that shows the number of equal parts taken of a whole quantity or unit. Some examples of fractions are $\frac{3}{4}$, $\frac{5}{8}$, $\frac{99}{100}$, and $\frac{17}{12}$. These same fractions written with a slash are $\frac{3}{4}$, $\frac{5}{8}$, $\frac{99}{100}$, and $\frac{17}{12}$.

The *denominator* of a fraction is the total number of equal parts in the whole quantity. The denominator is written below the bar.

The *numerator* of a fraction is the number that shows how many equal parts of the whole are taken. The numerator is written above the bar.

The numerator and denominator are called the *terms* of the fraction.

$\frac{3}{4}$ ← numerator
4 ← denominator

A *common fraction* consists of two whole numbers. $\frac{5}{5}$, $\frac{3}{7}$, and $\frac{13}{4}$ are all examples of common fractions.

A *proper fraction* has a numerator that is smaller than its denominator. Examples of proper fractions are $\frac{3}{4}$, $\frac{5}{11}$, $\frac{91}{92}$, and $\frac{247}{961}$.

An *improper fraction* is a fraction in which the numerator is larger than or equal to the denominator, as in $\frac{3}{2}$, $\frac{5}{4}$, $\frac{11}{8}$, $\frac{6}{6}$, and $\frac{17}{17}$.

A *mixed number* is a number composed of a whole number and a fraction, as in $3\frac{7}{8}$ and $7\frac{1}{2}$.

Note: $3\frac{7}{8}$ means $3 + \frac{7}{8}$. It is read as three and seven-eighths. $7\frac{1}{2}$ means $7 + \frac{1}{2}$. It is read as seven and one-half.

Writing fractions with a slash can cause people to misread a number. For example, some people might think that $1\frac{1}{4}$ means $1\frac{1}{4} = \frac{11}{4}$ rather than $1\frac{1}{4}$. For this reason, the slash notation for fractions will not be used in this book.

A *complex fraction* is a fraction in which one or both of the terms are fractions or mixed numbers, as in $\frac{\frac{3}{4}}{\frac{6}{15}}$, $\frac{32}{\frac{15}{4}}$, $\frac{8\frac{3}{4}}{3}$, $\frac{\frac{7}{16}}{\frac{2}{5}}$, and $\frac{4\frac{1}{4}}{7\frac{5}{8}}$.

EXPRESSING FRACTIONS AS EQUIVALENT FRACTIONS

The numerator and denominator of a fraction can be multiplied or divided by the same number without changing the value. For example, $\frac{1}{2} = \frac{1 \times 4}{2 \times 4} = \frac{4}{8}$. Both the numerator and denominator are multiplied by 4. Because $\frac{1}{2}$ and $\frac{4}{8}$ have the same value, they are *equivalent*. Also, $\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}$. Both numerator and denominator are divided by 4. Since $\frac{8}{12}$ and $\frac{2}{3}$ have the same value, they are equivalent. Equivalent fractions are necessary for comparing two fractions or for addition and subtraction of fractions.

A fraction is in its *lowest terms* when the numerator and denominator do not contain a common factor, as in $\frac{5}{9}$, $\frac{7}{8}$, $\frac{3}{4}$, $\frac{11}{12}$, $\frac{15}{32}$, and $\frac{9}{11}$. *Factors* are the numbers used in multiplying. For example, 2 and 5 are each factors of 10: $2 \times 5 = 10$. Expressing a fraction in lowest terms is often called *reducing* a fraction to lowest terms.

► **PROCEDURE** To reduce a fraction to lowest terms

- Divide both numerator and denominator by the greatest common factor (GCF).

Example Reduce $\frac{12}{42}$ to lowest terms.

Both terms can be divided by 2.
$$\frac{12 \div 2}{42 \div 2} = \frac{6}{21}$$

Note: The fraction is reduced, but not to lowest terms.

Further reduce $\frac{6}{21}$.

Both terms can be divided by 3.
$$\frac{6 \div 3}{21 \div 3} = \frac{2}{7} \text{ Ans}$$

Note: The value $\frac{2}{7}$ may be obtained in one step if each term of $\frac{12}{42}$ is divided by 2×3 , or 6. Six is the greatest common factor (GCF) of 12 and 42.

$$\frac{12 \div 6}{42 \div 6} = \frac{2}{7} \quad \text{Ans}$$

► **PROCEDURE** To express a fraction as an equivalent fraction with an indicated denominator that is larger than the denominator of the fraction

- Divide the indicated denominator by the denominator of the fraction.
- Multiply both the numerator and denominator of the fraction by the value obtained.

Example Express $\frac{3}{4}$ as an equivalent fraction with 12 as the denominator.

Divide 12 by 4.

$$12 \div 4 = 3$$

Multiply both 3 and 4 by 3.

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12} \quad \text{Ans}$$

EXPRESSING MIXED NUMBERS AS IMPROPER FRACTIONS

► **PROCEDURE** To express a mixed number as an improper fraction

- Multiply the whole number by the denominator.
- Add the numerator to obtain the numerator of the improper fraction.
- The denominator is the same as that of the original fraction.

Example 1 Express $4\frac{1}{2}$ as an improper fraction.

Multiply the whole number by the denominator.

$$4 \times 2 = 8$$

Add the numerator to obtain the numerator for the improper fraction.

$$8 + 1 = 9$$

The denominator is the same as that of the original fraction.

$$\frac{4 \times 2 + 1}{2} = \frac{9}{2} \quad \text{Ans}$$

Example 2 Express $12\frac{3}{16}$ as an improper fraction.

$$\frac{12 \times 16 + 3}{16} = \frac{195}{16} \quad \text{Ans}$$

EXPRESSING IMPROPER FRACTIONS AS MIXED NUMBERS

► **PROCEDURE** To express an improper fraction as a mixed number

- Divide the numerator by the denominator.
- Express the remainder as a fraction.

Examples Express the following improper fractions as mixed numbers.

$$\frac{11}{4} = 11 \div 4 = 2\frac{3}{4} \quad \text{Ans}$$

$$\frac{43}{3} = 43 \div 3 = 14\frac{1}{3} \quad \text{Ans}$$

$$\frac{931}{8} = 931 \div 8 = 116\frac{3}{8} \quad \text{Ans}$$

APPLICATION

Fractional Parts

1. Write the fractional part that each length, A through F, represents of the total shown on the scale in Figure 1-3.

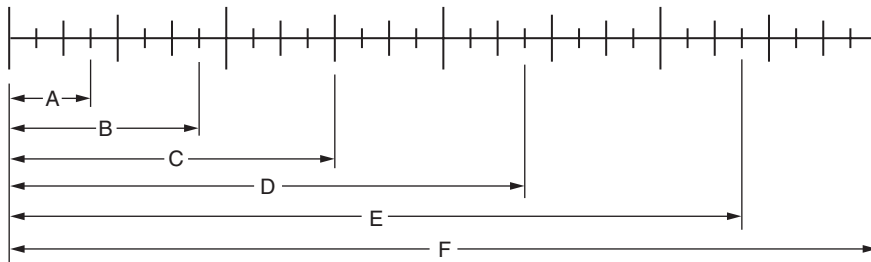


FIGURE 1-3

- A = _____
- B = _____
- C = _____
- D = _____
- E = _____
- F = _____

2. A welded support base is cut into four pieces as shown in Figure 1-4. What fractional part of the total length does each of the four pieces represent? All dimensions are in inches.

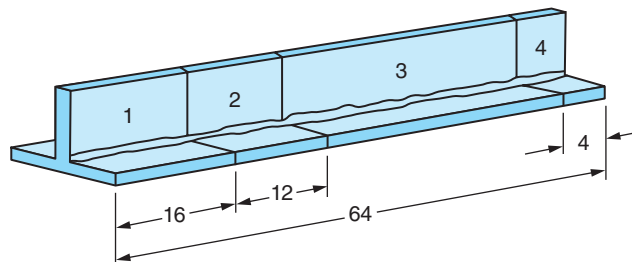


FIGURE 1-4

- Piece 1: _____
- Piece 2: _____
- Piece 3: _____
- Piece 4: _____

The circle in Figure 1-5 is divided into equal parts. Write the fractional part represented by each of the following in Exercises 3 and 4:

- 3. a. 1 part _____
- b. 3 parts _____
- c. 7 parts _____
- d. 5 parts _____
- e. 16 parts _____

- 4. a. $\frac{1}{2}$ of 1 part _____
- b. $\frac{1}{3}$ of 1 part _____
- c. $\frac{3}{4}$ of 1 part _____
- d. $\frac{1}{10}$ of 1 part _____
- e. $\frac{1}{16}$ of 1 part _____

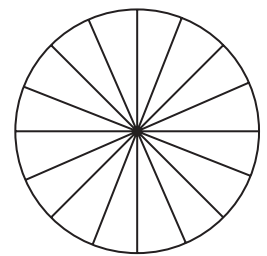


FIGURE 1-5

Expressing Fractions as Equivalent Fractions

5. Reduce to halves.

- a. $\frac{4}{8}$ _____
- b. $\frac{9}{18}$ _____
- c. $\frac{100}{200}$ _____
- d. $\frac{121}{242}$ _____

6. Reduce to halves.

- a. $\frac{25}{10}$ _____
- b. $\frac{18}{12}$ _____
- c. $\frac{126}{36}$ _____
- d. $\frac{225}{50}$ _____

Reduce numbers to lowest terms in Exercises 7 and 8.

7. a. $\frac{6}{8}$ _____

b. $\frac{12}{4}$ _____

c. $\frac{6}{10}$ _____

d. $\frac{30}{5}$ _____

e. $\frac{11}{44}$ _____

8. a. $\frac{14}{6}$ _____

b. $\frac{24}{8}$ _____

c. $\frac{65}{15}$ _____

d. $\frac{25}{150}$ _____

e. $\frac{14}{105}$ _____

Express the fractions in Exercises 9 and 10 as thirty-seconds.

9. a. $\frac{1}{4}$ _____

b. $\frac{3}{4}$ _____

c. $\frac{11}{8}$ _____

d. $\frac{7}{16}$ _____

10. a. $\frac{21}{16}$ _____

b. $\frac{19}{2}$ _____

c. $\frac{197}{16}$ _____

d. $\frac{21}{8}$ _____

In Exercises 11 and 12, express the given fractions as equivalent fractions with the indicated denominators.

11. a. $\frac{3}{4} = \frac{?}{8}$ _____

b. $\frac{7}{12} = \frac{?}{36}$ _____

c. $\frac{6}{15} = \frac{?}{60}$ _____

d. $\frac{17}{14} = \frac{?}{42}$ _____

e. $\frac{20}{9} = \frac{?}{45}$ _____

12. a. $\frac{14}{3} = \frac{?}{18}$ _____

b. $\frac{7}{16} = \frac{?}{128}$ _____

c. $\frac{13}{8} = \frac{?}{48}$ _____

d. $\frac{21}{16} = \frac{?}{160}$ _____

Mixed Numbers and Improper Fractions

Express the mixed numbers in Exercises 13 and 14 as improper fractions.

13. a. $2\frac{2}{3}$ _____

b. $1\frac{7}{8}$ _____

c. $5\frac{2}{5}$ _____

d. $3\frac{3}{8}$ _____

e. $5\frac{9}{32}$ _____

f. $8\frac{3}{7}$ _____

14. a. $10\frac{1}{3}$ _____

b. $9\frac{4}{5}$ _____

c. $100\frac{1}{2}$ _____

d. $4\frac{63}{64}$ _____

e. $49\frac{3}{8}$ _____

f. $408\frac{13}{16}$ _____

Express the improper fractions in Exercises 15 and 16 as mixed numbers.

15. a. $\frac{5}{3}$ _____
 b. $\frac{21}{2}$ _____
 c. $\frac{9}{8}$ _____
 d. $\frac{87}{4}$ _____
 e. $\frac{72}{9}$ _____
 f. $\frac{127}{124}$ _____

16. a. $\frac{127}{32}$ _____
 b. $\frac{57}{15}$ _____
 c. $\frac{150}{9}$ _____
 d. $\frac{235}{16}$ _____
 e. $\frac{514}{4}$ _____
 f. $\frac{401}{64}$ _____

Express the mixed numbers in Exercises 17 and 18 as improper fractions. Then express the improper fractions as the equivalent fractions indicated.

17. a. $2\frac{1}{2} = \frac{?}{8}$ _____
 b. $3\frac{3}{8} = \frac{?}{16}$ _____
 c. $7\frac{4}{5} = \frac{?}{15}$ _____

18. a. $12\frac{2}{3} = \frac{?}{18}$ _____
 b. $9\frac{7}{8} = \frac{?}{64}$ _____
 c. $15\frac{1}{2} = \frac{?}{128}$ _____

19. Sketch and redimension the plate shown in Figure 1-6. Reduce all proper fractions to lowest terms. Reduce all improper fractions to lowest terms and express as mixed numbers. All dimensions are in inches.

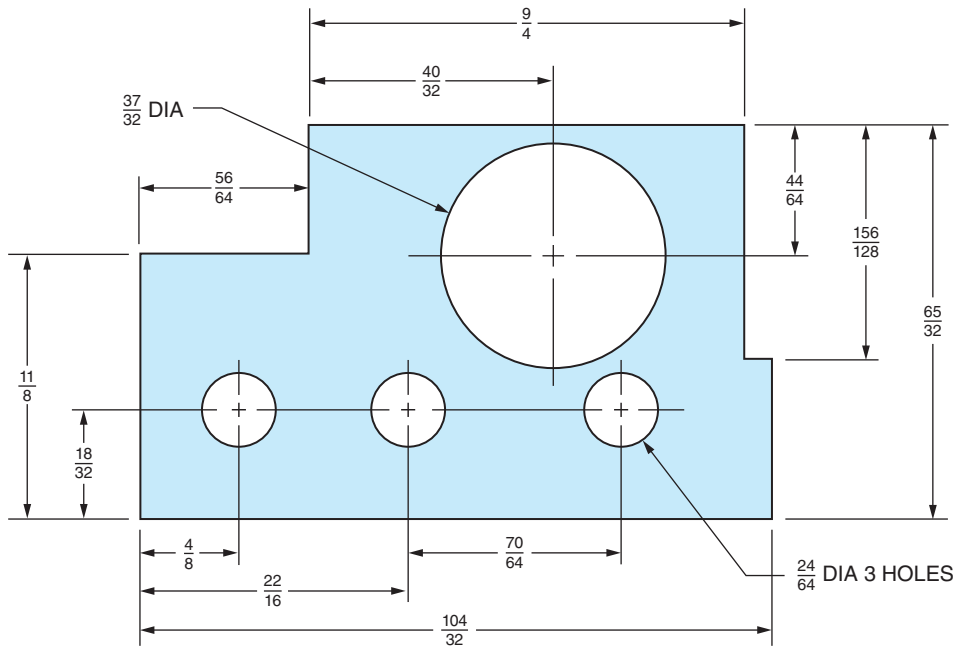


FIGURE 1-6

UNIT 2

Addition of Common Fractions and Mixed Numbers

OBJECTIVES After studying this unit you should be able to

- Determine lowest common denominators.
- Express fractions as equivalent fractions having lowest common denominators.
- Add fractions and mixed numbers.

A machinist must be able to add fractions and mixed numbers in order to determine the length of stock required for a job, the distances between various parts of a machined piece, and the depth of holes and cutouts in a workpiece.

LOWEST COMMON DENOMINATORS

Fractions cannot be added unless they have a common denominator. *Common denominator* means that the denominators of each of the fractions are the same, as in $\frac{5}{8}$, $\frac{7}{8}$, and $\frac{15}{8}$, which all have a common denominator of 8.

In order to add fractions that do not have common denominators, such as $\frac{3}{8} + \frac{1}{4} + \frac{7}{16}$, it is necessary to change to equivalent fractions with common denominators. Multiplying the denominators does give a common denominator, but it could be a very large number. We often find it easier to determine the lowest common denominator.

The *lowest common denominator* is the smallest denominator that is evenly divisible by each of the denominators of the fractions being added. Stated in another way, the *lowest common denominator* is the smallest denominator into which each denominator can be divided without leaving a remainder.

► **PROCEDURE** To find the lowest common denominator

- Determine the smallest number into which all denominators can be divided without leaving a remainder.
- Use this number as a common denominator.

Example 1 Find the lowest common denominator of $\frac{3}{8}$, $\frac{1}{4}$, and $\frac{7}{16}$.

The smallest number into which 8, 4, and 16 can be divided without leaving a remainder is 16. Write 16 as the lowest common denominator.

Example 2 Find the lowest common denominator of $\frac{3}{4}$, $\frac{1}{3}$, $\frac{7}{8}$, and $\frac{5}{12}$.

The smallest number into which 4, 3, 8, and 12 can be divided is 24.

The lowest common denominator is 24.

Note: In this example, denominators such as 48, 72, and 96 are common denominators because 4, 3, 8, and 12 divide evenly into these numbers, but they are not the lowest common denominators.

Although any common denominator can be used when adding fractions, it is generally easier and faster to use the lowest common denominator.

EXPRESSING FRACTIONS AS EQUIVALENT FRACTIONS WITH THE LOWEST COMMON DENOMINATOR

► **PROCEDURE** To change fractions into equivalent fractions having the lowest common denominator

- Divide the lowest common denominator by each denominator.
- Multiply both the numerator and denominator of each fraction by the value obtained.

Example 1 Express $\frac{2}{3}$, $\frac{7}{15}$, and $\frac{1}{2}$ as equivalent fractions having a lowest common denominator.

The lowest common denominator is 30. $30 \div 3 = 10$; $\frac{2 \times 10}{3 \times 10} = \frac{20}{30}$ Ans

Divide 30 by each denominator. $30 \div 15 = 2$; $\frac{7 \times 2}{15 \times 2} = \frac{14}{30}$ Ans

Multiply each term of the fraction by the value obtained. $30 \div 2 = 15$; $\frac{1 \times 15}{2 \times 15} = \frac{15}{30}$ Ans

Example 2 Change $\frac{5}{8}$, $\frac{15}{32}$, $\frac{3}{4}$, and $\frac{9}{16}$ to equivalent fractions having a lowest common denominator.

The lowest common denominator is 32.

$32 \div 8 = 4$; $\frac{5 \times 4}{8 \times 4} = \frac{20}{32}$ Ans $32 \div 4 = 8$; $\frac{3 \times 8}{4 \times 8} = \frac{24}{32}$ Ans

$32 \div 32 = 1$; $\frac{15 \times 1}{32 \times 1} = \frac{15}{32}$ Ans $32 \div 16 = 2$; $\frac{9 \times 2}{16 \times 2} = \frac{18}{32}$ Ans

ADDING FRACTIONS

► **PROCEDURE** To add fractions

- Express the fractions as equivalent fractions having the lowest common denominator.
- Add the numerators and write their sum over the lowest common denominator.
- Express an improper fraction as a mixed number when necessary and reduce the fractional part to lowest terms.